Can the flyby anomaly be attributed to earth-bound dark matter?

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We make preliminary estimates to assess whether the recently reported flyby anomaly can be attributed to dark matter interactions. We consider both elastic and exothermic inelastic scattering from dark matter constituents; for isotropic dark matter velocity distributions, the former decrease, while the latter increase, the final flyby velocity. The fact that the observed flyby velocity anomaly shows examples with both positive and negative signs, requires the dominance of different dark matter scattering processes along different flyby trajectories. The magnitude of the observed anomalies requires dark matter densities many orders of magnitude greater than the galactic halo density. Such a large density could result from an accumulation cascade, in which the solar system-bound dark matter density is much higher than the galactic halo density, and the earth-bound density is much higher than the solar system-bound density. We discuss a number of strong constraints on the hypothesis of a dark matter explanation for the flyby anomaly. These require dark matter to be non-self-annihilating, with the dark matter scattering cross section on nucleons much larger, and the dark matter mass much lighter, than usually assumed.

A. Introduction

In a recent paper, Anderson et al. [1] have reported anomalous orbital energy changes, of order 1 part in 10^6, during earth flybys of various spacecraft. Some flybys show energy decreases, and others energy increases, with the signs and magnitudes related to the spacecraft initial and final velocity orientation with respect to the equatorial plane. Since the DAMA/LIBRA collaboration [2] has recently reported an annual modulation signal interpreted as evidence for galactic halo dark matter, it is natural to ask whether the flyby anomalies could be attributed to dark matter interactions. In this paper we give some preliminary calculations directed at this question. Needless to say, in proceeding along this route we are assuming that the reported flyby anomalies are not artifacts of the orbital fitting method used in [1]. For a detailed discussion of this, and further references, see [3], which concludes that the most obvious candidates for artifactual explanations cannot give the large effect observed. This of course does not rule out the possibility that something

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has been overlooked, and searching for a conventional explanation of the flyby anomaly is clearly a line of investigation that should be vigorously pursued.[1]

B. Elastic and Inelastic Dark Matter Scattering

Let us consider the velocity change when a spacecraft nucleon[2] of mass $m_1 \simeq 1\text{GeV}$ and initial velocity $\vec{u}_1$ scatters from a primary dark matter particle of mass m_2 and initial velocity $\vec{u}_2$, into an outgoing nucleon of mass m_1 and velocity $\vec{v}_1$, and an outgoing secondary dark matter particle of mass $m_2' = m_2 - \Delta m$ and velocity $\vec{v}_2$. The inelastic case corresponds to $m_2' \neq m_2$, while in the elastic case, $m_2' = m_2$ and $\Delta m = 0$. (The possible relevance of inelastic scattering has been emphasized in a recent paper of Bernabei et al. [4]; see also the book of Khlopov [5], which gives arguments for unstable dark matter particles and reviews proposals [6] that dark matter may consist of "mirror" particles.) Under the assumptions, (i) both initial particles are nonrelativistic, so that $|\vec{u}_1| << c, |\vec{u}_2| << c$, and (ii) the center of mass scattering amplitude $f(\theta)$ depends only on the auxiliary polar angle θ of scattering[3], a straightforward calculation shows that the outgoing nucleon velocity change, averaged over scattering angles, is given by

$$\langle \delta \vec{v}_1 \rangle = \frac{m_2 \vec{u}_2 - m_2' \vec{u}_1}{m_1 + m_2'} + t \langle \cos\theta \rangle \frac{\vec{u}_1 - \vec{u}_2}{|\vec{u}_1 - \vec{u}_2|} \quad , \tag{1}$$

with $t > 0$ given by taking the square root of

$$t^2 = \frac{m_2 m_2'}{(m_1 + m_2)(m_1 + m_2')} (\vec{u}_1 - \vec{u}_2)^2 + \frac{\Delta m \; m_2'}{m_1 (m_1 + m_2')} \left[2c^2 - \frac{(m_1 \vec{u}_1 + m_2 \vec{u}_2)^2}{(m_1 + m_2)(m_1 + m_2')} \right] \quad , \tag{2}$$

and with $\langle \cos\theta \rangle$ given by

$$\langle \cos\theta \rangle = \frac{\int_0^\pi d\theta \sin\theta \cos\theta |f(\theta)|^2}{\int_0^\pi d\theta \sin\theta |f(\theta)|^2} \quad . \tag{3}$$

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[1] One possibility being discussed, and raised by a referee of this paper, is that the reported anomaly may arise from a mismodeling of the earth's reference frame within the barycentric system, since the earth's position relative to the sun is not known to a precision better than a kilometer. While it will be important to test the effects of this imprecision on integrations of the flyby trajectory, an argument based on energy conservation suggests that it will be too small. The magnitude of the change in the flyby potential energy per unit mass in the sun's gravitational field is $\sim GM_\odot \Delta R / A^2$, with $\Delta R \sim 1.4 \times 10^5\text{km}$ the distance travelled by the flyby between ingoing and outgoing asymptotes, with $A \sim 1.5 \times 10^8\text{km}$ the earth-sun distance, and with $GM_\odot \sim 1.3 \times 10^{11}\text{km}^3\text{s}^{-2}$. The error in this potential energy change arising from an uncertainty $\delta A \sim 1\text{km}$ in A is then $\sim 2GM_\odot \Delta R \delta A / A^3 \sim 10^{-8}\text{km}^2\text{s}^{-2}$. However, the magnitude of the anomaly in the flyby kinetic energy per unit mass is $\vec{v}_f \cdot \delta \vec{v}_f \sim 10^{-6} (10\,\text{km}\,\text{s}^{-1})^2 \sim 10^{-4}\text{km}^2\text{s}^{-2}$, which is four orders of magnitude larger than the error in the sun's potential energy arising from the uncertainty in the earth's position. A similar calculation for the moon shows that the $\sim$ 2cm uncertainty in its position relative to the earth makes a contribution eight orders of magnitude smaller than the flyby anomaly.

[2] Dark matter scattering from electrons would also be expected, with subsequent sharing of the momentum change with nucleons, but since this is harder to model, we ignore it for the purpose of making order of magnitude estimates.

[3] Here θ is the kinematically free angle between $\vec{v}_1 - (m_1 \vec{u}_1 + m_2 \vec{u}_2)/(m_1 + m_2')$ and $\vec{u}_1 - (m_1 \vec{u}_1 + m_2 \vec{u}_2)/(m_1 + m_2)$, which when $m_2' \neq m_2$ is not the same as the angle between the incident and outgoing nucleon in the center of mass frame.

In the elastic scattering case, with $\Delta m = 0$, $m_2' = m_2$, these equations simplify to

$$\langle \delta \vec{v}_1 \rangle = -2 \frac{m_2}{m_1 + m_2} (\vec{u}_1 - \vec{u}_2) \langle \sin^2(\theta/2) \rangle \quad . \tag{4}$$

In the inelastic case, assuming that $\Delta m / m_2$ and m_2'/m_2 are both of order unity, the equations are well approximated by

$$\langle \delta \vec{v}_1 \rangle \simeq \frac{\vec{u}_1 - \vec{u}_2}{|\vec{u}_1 - \vec{u}_2|} \left(\frac{2 \Delta m \; m_2'}{m_1 (m_1 + m_2')} \right)^{1/2} c \langle \cos \theta \rangle \quad . \tag{5}$$

Since $\vec{u}_1$ and $\vec{u}_2$ are typically of order 10 km s^{-1}, the velocity change in the inelastic case is larger than that in the elastic case by a factor $\sim c/|\vec{u}_1| \sim 10^4$.

To get the force per unit spacecraft mass resulting from dark matter scatters, that, is the acceleration, one multiplies the velocity change in a single scatter $\langle \delta \vec{v}_1 \rangle$ by the number of scatters per unit time. This latter is given by the flux $|\vec{u}_1 - \vec{u}_2|$, times the scattering cross section σ, times the dark matter spatial and velocity distribution $\rho\big(\vec{x}, \vec{u}_2\big)$. Integrating out the dark matter velocity, one thus gets for the force acting at the point $\vec{x}(t)$ on the spacecraft trajectory with velocity $\vec{u}_1 = d\vec{x}(t)/dt$,

$$\delta \vec{F} = \int d^3 u_2 \langle \delta \vec{v}_1 \rangle |\vec{u}_1 - \vec{u}_2| \sigma \rho\big(\vec{x}, \vec{u}_2\big) \quad . \tag{6}$$

Equating the work per unit spacecraft mass along a trajectory from t_i to t_f to the change in kinetic energy per unit mass (assuming that the initial and final times are in the asymptotic region where the potential energy can be neglected) we get

$$\begin{aligned} \delta \frac{1}{2} (\vec{v}_f^{\,2} - \vec{v}_i^{\,2}) = & \vec{v}_f \cdot \delta \vec{v}_f = \int_{t_i}^{t_f} dt (d\vec{x}/dt) \cdot \delta \vec{F} \\ = & \int_{t_i}^{t_f} dt \int d^3 u_2 (d\vec{x}/dt) \cdot \langle \delta \vec{v}_1 \rangle |\vec{u}_1 - \vec{u}_2| \sigma \rho\big(\vec{x}, \vec{u}_2\big) \quad . \end{aligned} \tag{7}$$

To get the vectorial change in velocity is more difficult; one must solve the perturbed orbital differential equation (taking here the center of the earth as the origin of coordinates),

$$\frac{d^2 \delta \vec{x}}{(dt)^2} = -\frac{G M_\oplus}{|\vec{x}|^3} \left(\delta \vec{x} - \frac{3 \vec{x} \cdot \delta \vec{x} \, \vec{x}}{|\vec{x}|^2} \right) + \delta \vec{F} \quad . \tag{8}$$

One can check that taking the inner product of this equation with $d\vec{x}/dt$, integrating over time, and integrating by parts twice, again gives the energy conservation relation $\vec{v}_f \cdot \delta \vec{v}_f = \int_{t_i}^{t_f} dt d\vec{x}/dt \cdot \delta \vec{F}$.

Consider now the case of a dark matter density that has an inversion invariant velocity distribution, so that $\rho\big(\vec{x}, \vec{u}_2\big) = \rho\big(\vec{x}, -\vec{u}_2\big)$. From (4) and (7), we see that in the elastic case, the flux weighting factor favors $\vec{u}_2$ being oppositely directed to $\vec{u}_1 = d\vec{x}(t)/dt$, and so the flyby velocity change, integrated over the dark matter velocity distribution, is oppositely directed to $d\vec{x}(t)/dt$. Hence, as expected for elastic scattering, one gets a positive drag coefficient and the net effect is a reduction in spacecraft velocity. Turning to the inelastic case, where the flux factor in (7) cancels the denominator $|\vec{u}_1 - \vec{u}_2|$ in (5), the integration over $\vec{u}_2$ leaves only the term $\vec{u}_1 = d\vec{x}(t)/dt$, and so in this case the flyby velocity change, integrated over the dark matter velocity distribution, is parallel to $d\vec{x}(t)/dt$ when $\langle \cos\theta \rangle > 0$. So for forward dominated exothermic inelastic scattering, the drag coefficient is negative and the net effect is an increase in spacecraft velocity, while for backward dominated inelastic scattering, the drag coefficient is positive, as in the elastic case. Since the observations reported in [1] show cases of increased velocity, and of decreased velocity, a dark matter explanation (assuming an approximately isotropic velocity distribution) requires the presence, in differing proportions on different trajectories, of inelastic forward dominated scattering, and of either elastic or inelastic backward dominated scattering. This could be achieved in a two-component dark matter model, with differing spatial densities $\rho(\vec{x}, \vec{u}_2)$ governing the inelastic and elastic scatterers. Another possibility is a single dark matter component with an anisotropic velocity distribution, undergoing inelastic scattering, and possibly also elastic scattering as well. Detailed modelling will be needed to see which possibilities are viable.

C. Quantitative estimates

Let us now turn to some quantitative estimates. To get a velocity change of order 10^{-6} of the spacecraft velocity over a time interval T one needs

$$10^{-6} \sim T \bar{f} \sigma \bar{\rho} |\langle \delta \vec{v}_1 \rangle| / |\vec{v}_f| \quad , \tag{9}$$

with $\bar{f}$ the average flux, $\bar{\rho}$ the average dark matter density, σ the scattering cross section, and $|\langle \delta \vec{v}_1 \rangle|$ the magnitude of the single scattering velocity changes given, in the elastic and inelastic cases, by (4) and (5) respectively. This gives an estimate of the required product of mean dark matter density times interaction cross section,

$$\sigma \bar{\rho} \sim 10^{-6} |\vec{v}_f| / (T \bar{f} |\langle \delta \vec{v}_1 \rangle|) \quad . \tag{10}$$

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Anderson et al. [1] report that for the NEAR spacecraft flyby, the velocity change occurs during an interval $T = 3.7\text{h} \sim 10^4\text{s}$ when the spacecraft could not be tracked during near earth approach.

Taking this estimate for T and taking the mean flux as $\bar{f} \sim 10\text{km s}^{-1} = 10^6\text{cm s}^{-1}$, (10) becomes

$$\sigma\bar{\rho} \sim 10^{-16}\text{cm}^{-1}|\vec{v}_f|/|\langle\delta\vec{v}_1\rangle| \quad . \tag{11}$$

Defining the mean dark matter mass density as $\bar{\rho}_m = m_2\bar{\rho}$, using (4) gives for the elastic case

$$\sigma\bar{\rho}_m \sim 10^{-16}\text{cm}^{-1}(m_1 + m_2) \geq 10^{-16}(\text{GeV}/c^2)\text{cm}^{-1} \quad , \tag{12}$$

while using (5) gives for the inelastic case

$$\sigma(\Delta m m_2')^{1/2}\bar{\rho} \sim \sigma\bar{\rho}_m \sim 10^{-20}\text{cm}^{-1}[m_1(m_1 + m_2')]^{1/2} \geq 10^{-20}(\text{GeV}/c^2)\text{cm}^{-1} \quad . \tag{13}$$

To estimate dark matter densities from these bounds, we must assume a value for the scattering cross section. For a cross section of order 1 picobarn $=10^{-36}\text{cm}^2$, we get dark matter mass densities $\bar{\rho}_m \sim 10^{20}(\text{GeV}/c^2)\,\text{cm}^{-3}$ in the elastic case, and $\bar{\rho}_m \sim 10^{16}(\text{GeV}/c^2)\,\text{cm}^{-3}$ in the inelastic case, respectively. For a cross section of order 1 millibarn $=10^{-27}\text{cm}^2$, which would require dark matter masses much below a GeV, we get corresponding dark matter mass densities $\bar{\rho}_m \sim 10^{11}(\text{GeV}/c^2)\,\text{cm}^{-3}$ in the elastic case, and $\bar{\rho}_m \sim 10^{7}(\text{GeV}/c^2)\,\text{cm}^{-3}$ in the inelastic case, respectively. These dark matter mass density bounds are orders of magnitudes larger than the estimated galactic halo dark matter mass density of $0.3(\text{GeV}/c^2)\text{cm}^{-3}$, but are still many orders of magnitude smaller than the earth mass density of about $3 \times 10^{24}(\text{GeV}/c^2)\text{cm}^{-3}$.

Can such large dark matter densities exist in orbit around the earth? In a separate note [7], we have pointed out that by comparing the total mass (in gravitational units) of the earth-moon system, as determined by lunar laser ranging, with the sum of the lunar mass as independently determined by its gravitational action on satellites or asteroids, and the earth mass as determined by the LAGEOS geodetic survey satellite, one can get a direct measure of the mass of earth-bound dark matter lying between the radius of the moon's orbit and the geodetic satellite orbit. Current data show that the mass of such earth-bound dark matter must be less than 4×10^{-9} of the earth's mass, giving an upper dark matter mass limit of $1.3 \times 10^{43}\text{GeV}/c^2$. To explain the flyby anomalies, earth-bound dark matter would have to be concentrated within a radius of about 70,000 km around earth, which contains a volume of $\simeq 1.4 \times 10^{30}\text{cm}^3$; for the dark matter mass within this volume not to exceed $4 \times 10^{-9} M_{\oplus}$, the mean dark matter density would have to be bounded by about $10^{13}(\text{GeV}/c^2)\text{cm}^{-3}$. By the above estimates, this would correspond, in the inelastic case, to a cross section $\sigma > 10^{-33}\text{cm}^2$, and in the elastic case, to a cross section $\sigma > 10^{-29}\text{cm}^2$. These cross sections are much larger than usually assumed for the interactions of dark matter with nucleons, but can be compatible with existing bounds on dark matter interaction cross sections if the dark matter mass is much below a GeV.

D. Accumulation cascade

Because earth-bound dark matter mass densities of order $\bar{\rho}_m \sim 10^7(\mathrm{GeV}/c^2)\mathrm{cm}^{-3}$ or larger greatly exceed the estimated galactic halo dark matter mass density, a mechanism for concentrating dark matter near earth would be needed. One possibility is an accumulation cascade, in which solar system-bound dark matter is accumulated over the lifetime of the solar system, and then this enhanced dark matter density leads to a further accumulation near earth. Bearing in mind that it is an open question whether there are efficient mechanisms for dark matter capture by the solar system or earth [8], [9], [10] we nonetheless proceed to estimate whether such a mechanism, with a high capture fraction, could lead to the dark matter densities needed to explain the flyby discrepancies. We note also that Frère, Ling, and Vertongen [11] have pointed out that local dark matter concentrations in the galaxy may have played a role in the formation of the solar system, which could give another mechanism for producing a higher sun-bound or earth-bound dark matter density than the mean galactic halo density.

Let us start with the solar system, which is moving through the galaxy at a velocity of $v_{\mathrm{s.s.}} \sim 220\mathrm{km\ s}^{-1}$, with the local galactic halo dark matter approximated by a Maxwellian velocity distribution with a r.m.s. velocity of similar magnitude. Let $f_{\mathrm{s.s.}}$ be the probability of capture of a dark matter particle near a solar system earth orbit of radius $A \equiv 1\mathrm{a.u.} \simeq 1.5 \times 10^8\mathrm{km}$. Then assuming particles captured in an annulus of radius A and area $2\pi A dA$ over the solar system lifetime $T_{\mathrm{s.s.}} \sim 1.5 \times 10^{17}\mathrm{s}$ are redistributed, over time, into a volume $4\pi A^2 dA$, the captured particle mass density at radius A would be

$$\rho_{m;\mathrm{s.s.}}/\rho_{m;\mathrm{halo}} \sim \frac{f_{\mathrm{s.s.}}}{2A} v_{\mathrm{s.s.}} T_{\mathrm{s.s.}} \sim 10^{11} f_{\mathrm{s.s.}} \quad . \tag{14}$$

So for $f_{\mathrm{s.s.}}$ of unity, a very large concentration of dark matter particles in the solar system would be possible. In fact, the known limits on a local excess of solar system dark matter [11], [12] are about 3×10^5 times the galactic halo mass density, so $f_{\mathrm{s.s.}}$ in (14) could be of order 10^{-5} at most. We remark in passing that an enhanced solar system density of dark matter particles would show up as a daily sidereal time modulation of dark matter particle counting rates in sufficiently sensitive experiments of the DAMA/LIBRA type, just as the galactic halo dark matter density is detected by DAMA/LIBRA as an annual modulation [13] in the counting rate.

Given an enhanced solar system dark matter density, we can now make a similar estimate of the maximum possible capture density in an earth orbit, by replacing $f_{\mathrm{s.s}}$ by the corresponding earth capture fraction f_{e}, replacing $v_{\mathrm{s.s.}}$ by the orbital velocity of earth around the sun $v_{\mathrm{e}} \sim 30\mathrm{km\ s}^{-1}$, and replacing A by the earth orbit radius relevant for the flyby anomalies, $R \sim 7 \times 10^4$km. This gives

$$\rho_{m;\mathrm{e}}/\rho_{m;\mathrm{s.s.}} \sim \frac{f_{\mathrm{e}}}{4R} v_{\mathrm{e}} T_{\mathrm{s.s.}} \sim 2 \times 10^{13} f_{\mathrm{e}} \quad , \tag{15}$$

where we have divided by an extra factor of 2 since we are assuming that the solar system dark matter density is linearly increasing over its lifetime. So if the solar system dark matter density were equal to its upper bound, and f_{e} were of order unity, the earth-bound dark matter density at or below the radius relevant for the flyby anomalies could be as large as $\sim 10^{19}$ times the galactic halo density. So even with small values of f_{e}, one could attain large enough values of dark matter density to explain the flyby discrepancy if the interaction cross section were large enough.

E. Constraints

In addition to having to provide a large enough dark matter density, such a mechanism would have to lead to a dark matter spatial distribution satisfying significant constraints. We shall consider three types of constraints, (1) constraints coming from data on closed orbits of satellites, the moon, and the earth, and (2) constraints coming from stellar dynamics, and (3) constraints coming from earth and satellite heating.

1. Closed orbit constraints

We begin with an analysis of closed orbit constraints, by asking what is the most general form of a drag force that gives zero cumulative drag for all closed satellite orbits. Let us rewrite (7) for the work per unit spacecraft mass as

$$\delta W = \int dt (d\vec{x}/dt) \cdot \delta \vec{F} = \int d\theta (d\vec{x}/d\theta) \cdot \delta \vec{F} \quad , \tag{16}$$

with θ the angle in the orbital plane between the orbit semi-major axis and the vector from the earth's center to the satellite, and let us define the "drag function" $D(\vec{x}, \vec{v} = d\vec{x}/dt)$ as

$$D(\vec{x}, \vec{v}) = (d\vec{x}/d\theta) \cdot \delta \vec{F} \quad . \tag{17}$$

Then the condition for vanishing cumulative drag over the orbit becomes

$$\int_0^{2\pi} d\theta D\big(\vec{x}(\theta), \vec{v}(\theta)\big) = 0 \quad . \tag{18}$$

Since each pair $\vec{x}, \vec{v}$ is Cauchy data that corresponds to a distinct orbit, a general solution to (18) is

$$D(\vec{x}, \vec{v}) = \sum_{\ell=1}^{\infty} (a_\ell \sin \ell\theta + b_\ell \cos \ell\theta) \quad , \tag{19}$$

with θ determined by $\vec{x}, \vec{v}$ and with the coefficients a_ℓ, b_ℓ functions of the five orbit constants of motion (angular momentum vector, energy, and semi-major axis orientation) that in turn can be computed as functions of $\vec{x}$, $\vec{v}$.[4] That is, (18) is satisfied by requiring that the Fourier series expansion in θ of the drag function has no constant term b_0. For a hyperbolic orbit such as the flyby orbits, the cumulative energy change per unit spacecraft mass is obtained by integrating (16) from $-\theta_D$ to θ_D, with $2\theta_D$ the flyby deflection angle, giving

$$\delta \frac{1}{2}(\vec{v}_f^2 - \vec{v}_i^2) = 2b_0\theta_D + 2\sum_{\ell=1}^{\infty} \frac{b_\ell}{\ell} \sin \ell\theta_D \quad , \tag{20}$$

where we have included the possibility of a nonzero b_0. Details of how the near-earth environment (such as a hypothetical dark matter distribution) influence the flyby energy change appear through the coefficient functions b_ℓ. In particular, the fitting formula given in [1] would have to arise this way, through the dynamics determining the coefficients b_ℓ, and not through the kinematics of requiring vanishing drag anomaly for closed orbits, corresponding to vanishing b_0.[5]

This analysis suggests that if the flyby effect is confirmed, there likely will be analogous drag anomalies in high-lying satellite orbits, since a vanishing b_0 would require a “fine-tuning” in the drag law, with cancelling negative and positive drag contributions around closed orbits. Since normal satellite atmospheric drag effects are proportional to the cross-sectional area of the satellite, whereas dark matter scattering drag (of either sign) is proportional to the mass of the satellite, it would be helpful to have an analysis of drag effects in existing satellites, assuming the presence of both area-proportional and mass-proportional components. The aim would be to see if there is any evidence for small mass-proportional drag contributions, or at least to place bounds on such contributions for use as constraints on dark matter model fits to the flyby data.

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[4] This statement applies to $\vec{x}, \vec{v}$ values that do not correspond to earth-intersecting orbits.

[5] For example, one might try a dynamical model in which there are inelastic and elastic scatterers with roughly similar values of scattering cross section times density, with the inelastic scatterers distributed in a prolate ellipsoid, elongated towards the poles, and the elastic scatterers in a oblate ellipsoid, somewhat elongated towards the equator. For circular satellite orbits in the overlap region of the two distributions, the positive and negative drag effects would cancel; for a flyby deflected from a small to a large angle with respect to the equator, the negative drag effects would predominate, giving a velocity increase. For a flyby deflected from a moderate to a smaller angle with respect to the equator, the positive drag effects would dominate, giving a velocity decrease. Such a model would not reproduce the fitting formula given in [1], but with appropriate shapes of the density profiles might be able to accommodate the six flyby data sets used to generate that fit. Clearly, this is but one example of many possible scenarios.

Ignoring now the possibility of fine-tuning of the drag force that could give cancellation between negative and positive contributions over closed orbits, let us analyze several constraints that come from observation of the rate at which the radius of an orbiting body increases or decreases, which can be used to bound a drag force acting on it as follows. For definiteness let us consider the earth's orbit around the sun, since other cases can be obtained from this by appropriate substitutions. Approximating the earth's orbit as circular, the total energy (potential plus kinetic) is $E = -GM_{\oplus}M_{\odot}/(2A)$, and the orbital velocity is given by $v_e^2 = GM_{\odot}/A$, from which one easily derives that

$$\frac{dE}{M_{\oplus}c^2} = \frac{1}{2}\frac{v_e^2}{c^2}\frac{dA}{A} \quad . \tag{21}$$

Letting dA be the change in A over a single orbit, the left hand side of (21) is given by $2\pi A$ times the force per unit mass-energy, which by use of (5) for the inelastic case and (6) is given by

$$2\pi A(m_2/m_1)(v_e/c)\sigma\bar{\rho}_{\rm s.s.} \quad , \tag{22}$$

with $\bar{\rho}_{\rm s.s.}$ the mean density of sun-bound dark matter along the earth's orbit. Again writing $\rho_{m;{\rm s.s.}} = m_2\rho_{\rm s.s.}$, from (21) and (22) we get the relation

$$\sigma\bar{\rho}_{m;{\rm s.s.}} = \frac{1}{4\pi}\frac{dA}{A^2}\frac{v_e}{c}m_1 \quad . \tag{23}$$

Taking for dA the uncertainty in the change in A over one orbit, this gives a bound on the product $\sigma\bar{\rho}_m$ acting over the orbit.

If an inelastic earth-bound dark matter scattering mechanism is responsible for the negative drag flyby anomalies, then the earth's motion through sun-bound dark matter will produce acceleration anomalies in the earth's orbit, which (assuming no fine-tuning cancellations) can be used to place a bound on the density of sun-bound dark matter. Current bounds on the yearly change in the radius $A \sim 1.5 \times 10^8$km of the earth's orbit are around 1.5 m per century [14], or $dA \sim 1.5$cm per orbit. Substituting this, the value of A, and $v_e \sim 30{\rm kms}^{-1}$ into (23) gives the bound

$$\sigma\bar{\rho}_{m;{\rm s.s.}} \leq 0.5 \times 10^{-31}({\rm GeV}/c^2){\rm cm}^{-1} \quad . \tag{24}$$

However, this formula must be corrected to take into account the fact that for cross sections $\sigma > 10^{-33}{\rm cm}^2$, the earth diameter exceeds the optical depth for dark matter scattering on nucleons, and so not all nucleons in the earth have an equal probability of undergoing a dark matter scattering. Letting $F_{\rm e}$ denote the participating fraction of earth nucleons, (24) must be modified to read

$$\sigma\bar{\rho}_{m;{\rm s.s.}}F_{\rm e} \leq 0.5 \times 10^{-31}({\rm GeV}/c^2){\rm cm}^{-1} \quad . \tag{25}$$

In terms of the density of nucleons in earth $\rho_{\rm earth} \sim 3.3 \times 10^{24} {\rm cm}^{-3}$ and the earth diameter $D_{\rm earth} \sim 1.3 \times 10^9 {\rm cm}$, , an estimate of F is

$$F_{\rm e} \sim \frac{1}{\rho_{\rm earth} D_{\rm earth} \sigma} \sim \frac{0.2 \times 10^{-33} {\rm cm}^2}{\sigma} \quad . \tag{26}$$

When substituted into (25), this gives the bound

$$\bar\rho_{m;{\rm s.s.}} \leq 2 \times 10^2 ({\rm GeV}/c^2) {\rm cm}^{-3} \quad . \tag{27}$$

This bound (which we emphasize depends on the hypothesis of an inelastic dark matter collision explanation for the flyby anomaly, and assumes no cancellation of negative and positive drag effects for the earth orbit) is considerably lower than the current limit of $\sim 10^5 ({\rm GeV}/c^2) {\rm cm}^{-3}$ on excess solar system dark matter. Taking the ratio of (13), which refers to $\bar\rho_{m;{\rm e}}$, to (27), and comparing with (15), we learn that the earth capture fraction in the cascade scenario must obey the constraint

$$f_e \geq \frac{0.2 \times 10^{-35} {\rm cm}^2}{\sigma} \quad . \tag{28}$$

For $\sigma = 10^{-33} {\rm cm}^2$, this requires the relatively large earth capture fraction $f_e \geq 0.2 \times 10^{-2}$, but for larger values of σ the requirement on f_e becomes less stringent; for example, for $\sigma = 10^{-27} {\rm cm}^2$, (28) becomes $f_e \geq 0.2 \times 10^{-8}$.

We next apply (23) to the moon's orbit around the earth. Lunar ranging [15] has established the position of the moon to within a post-fit residual accuracy of about 2 cm relative to an earth–moon distance of $A_m = 384,000 {\rm km}$. The moon is found to be receding from the earth at a rate of 3.8cm per year, or 0.28 cm per orbit, which is explained by the action of tidal effects. Estimating the uncertainty in this as $dA_m \sim 0.07\,{\rm cm}$ for one orbit, and substituting this, the moon's orbital velocity $v_m \sim 1\,{\rm km\,s}^{-1}$, and the moon's orbit radius A_m into (23), we find that along the moon's orbit we must have

$$\sigma \bar\rho_{m;{\rm e}} < \frac{1}{4\pi} \frac{dA_m}{A_m^2} \frac{v_m}{c} m_1 \sim 10^{-29} ({\rm GeV}/c^2) {\rm cm}^{-1} \quad . \tag{29}$$

However, since the radius of the moon is about 0.3 that of the earth, and the density of the moon is about 0.6 that of earth, for cross sections $\sigma > 6 \times 10^{-33} {\rm cm}^2$, a correction for optical depth $\sim 6F_{\rm e} \sim 10^{-33} {\rm cm}^2/\sigma$ is again needed. Following the reasoning of (25) through (27), we end up with the constraint

$$\bar\rho_{m;{\rm e}} \leq 10^4 ({\rm GeV}/c^2) {\rm cm}^{-3} \quad . \tag{30}$$

Hence the earth-bound dark matter density at the orbit of the moon would have to be many orders of magnitude smaller than the dark matter density within the radius of 70,000 km relevant for the flyby anomaly. For example, for an inelastic cross section $\sigma \sim 10^{-28}\mathrm{cm}^2$, the dark matter density at the moon's orbit would have to be 10^{-4} of that needed to explain the flyby anomaly, while for an inelastic cross section of $10^{-32}\mathrm{cm}^2$ it would have to be a factor 10^{-8} smaller.

There are also low altitude constraints coming from considering satellite orbits. The satellites of the global positioning system have orbit radius of 26,600 km, and geosynchronous satellites have orbit radii of $\sim 42,000$ km, but the orbits of these satellites have not been monitored to the level of precision [16] of that of the LAGEOS geodetic satellite [17], with orbit radius of $\sim 12,300$ km. Residual accelerations of the LAGEOS satellite, believed to arise from drag effects related to crossings of the earth's shadow, are smaller in magnitude than $\sim 3 \times 10^{-12}\mathrm{ms}^{-2}$, as compared with the anomalous flyby accelerations $\sim 10^{-6}10^4\mathrm{ms}^{-1}/10^4\mathrm{s} = 10^{-6}\mathrm{ms}^{-2}$. Thus, dark matter densities at the radius of the LAGEOS orbit would have to be smaller by a factor of 3×10^{-6} than at the orbit radii relevant for the flyby anomaly, corresponding to a constraint, in the inelastic case,

$$\sigma \bar{\rho}_{m;\mathrm{e}} \leq 3 \times 10^{-26}(\mathrm{GeV}/c^2)\mathrm{cm}^{-1} \quad . \tag{31}$$

It would clearly be of interest to have comparable anomalous acceleration limits for the higher-orbiting global positioning system and geosynchronous satellites, since these come closer to the radius 70,000 km relevant for the flyby anomaly.

We consider finally what the comparable figure would be for Phobos, the moon which orbits Mars with an orbital radius of $\sim 9,400$km, with an orbital period $\sim$ 7h40m, an orbital velocity of $\sim 2.1\mathrm{km\,s}^{-1}$, and an orbital radius decay of $1.8\mathrm{cm\,y}^{-1}$. Application of (29) to this case, assuming that the residual uncertainty in the orbital decay after taking account of tidal effects is approximately 1/4 of the measured value, gives an upper bound to the dark matter density at the Phobos orbit

$$\sigma \bar{\rho}_m < 2 \times 10^{-28}(\mathrm{GeV}/c^2)\mathrm{cm}^{-1} \quad , \tag{32}$$

which is a factor of 100 tighter than the LAGEOS bound of (31). Since ρ_m near Phobos is necessarily greater than the galactic halo density of $0.3(\mathrm{GeV}/c^2)\mathrm{cm}^{-1}$, this implies that the inelastic cross section is bounded by $\sigma < 7 \times 10^{-28}\mathrm{cm}^2$. Conversely, since we have inferred from the limit on total earth-bound dark matter that $\sigma > 10^{-33}\mathrm{cm}^2$, we also learn that near Phobos we must have $\rho_{m;\mathrm{s.s.}} < 2 \times 10^5(\mathrm{GeV}/c^2)\mathrm{cm}^{-3}$, consistent with known limits on solar system-bound dark matter.

2. Stellar (and solar) dynamics constraints

Other possible problems raised by postulating a sun-bound dark matter density larger than the galactic halo density are whether the resulting dark matter accretion on the sun (i) exceeds the uncertainty in the loss of solar mass from radiation and solar wind, and (ii) unacceptably alters our well-understood model of solar dynamics. The second of these has been discussed in detail, through a running of stellar dynamics codes including dark matter capture, in a recent paper of Fairbairn, Scott and Edsjö [18], and concludes that "for a spin-dependent WIMP-nucleon cross section of $\sigma = 10^{-38}\mathrm{cm}^2$, stars only start to change their behavior when immersed in a dark matter density of around 10^8 or 10^9 GeV cm^{-3}", which corresponds to $\sigma\rho_{m\odot} \sim 10^{-30}$ to $10^{-29}(\mathrm{GeV}/c^2)\mathrm{cm}^{-1}$, with $\rho_{m\odot}$ the dark matter density near the sun. To convert to a limit on $\bar{\rho}_{m;s.s.}$, which we have defined as the density of sun-bound dark matter near the earth's orbit, we should take account of the fact that the sun-bound dark matter density near the sun may be higher than that near the earth's orbit. Dividing by a factor of $A/R_\odot = 1.5 \times 10^8/7 \times 10^5 = 214$, with $R_\odot$ the solar radius, as suggested by (14), we can write the constraint coming from [18] as

$$\bar{\rho}_{m;s.s.} \leq \frac{10^{-33}\mathrm{cm}^2}{\sigma}(5 \text{ to } 50)(\mathrm{GeV/c^2})\mathrm{cm}^{-3} \quad , \tag{33}$$

which is a highly restrictive limit on the density of sun-bound dark matter. However, this limit assumes the case of self-annihilating dark matter discussed in [18]; for non-self-annihilating dark matter, as we consider below in discussing earth-capture constraints, this sun-capture restriction can be substantially weakened by a factor $\sim 10^5$, and should pose no problem.[6] For the potential problem (i), one can use the dark matter mass capture rate formula[7]

$$\dot{M} \sim \sigma\rho_{m;\odot}(M_\odot/m_1)(v_{\mathrm{esc}}^2/v_{\mathrm{dm}}) \quad , \tag{34}$$

with $v_{\mathrm{esc}} \sim 620\,\mathrm{km\,s^{-1}}$ the escape velocity from the sun and with $v_{\mathrm{dm}} \sim 300\,\mathrm{km\,s^{-1}}$ the velocity of dark matter near the sun. Using the bound $\sigma\rho_{m\odot} \leq 10^{-29}(\mathrm{GeV}/c^2)\mathrm{cm}^{-1}$ inferred above from [18], this evaluates to $\dot{M} \sim 4 \times 10^{-14} M_\odot \mathrm{y}^{-1}$, which is smaller than the estimated rate [14] of solar mass loss from radiation and solar wind, of $\sim 9 \times 10^{-14} M_\odot \mathrm{y}^{-1}$, and of the same order as the uncertainties in this rate.

3. Earth and satellite heating constraints

We consider next constraints coming from earth and satellite heating, which we shall see are sensitive to the value of the dark matter particle mass m_2. In making these estimates, we as-

[6] I wish to thank Pat Scott for an email pointing out that the stellar dynamics constraint likely requires that we assume the sun-bound dark matter to be non-self-annihilating, and for bringing the paper [18] to my attention. It would be interesting to know the limits analogous to those of [18] for various cases of non-self-annihilating dark matter.

[7] This is eq. (11) of [19], which is a simplification of (2.31) of Gould [20]; when the dark matter mass is much larger or smaller than the nucleon mass, the capture rate is smaller than this estimate.

sume that the dark matter is not self-annihilating; however, the self-annihilating case would give estimates similar to the first case discussed below. We shall consider bounds on the dark matter density near the earth's surface $\rho_{m;R_\oplus}$ following [21] from the earth's heat flow budget.[8] Let us focus on the inelastic case, and suppose that the cross section for the primary dark matter particle of mass m_2 to inelastically scatter on a nucleon into the secondary dark matter particle of mass m_2' is larger than $10^{-33}\,\mathrm{cm}^2$. In this case, the optical depth of the earth is less than one earth diameter, and an appreciable fraction of earth-intersecting primary dark matter particles will interact. There are then two limiting cases to consider. In the first case, the cross section for interaction of the secondary dark matter particle of mass m_2' is also large enough for this particle to be trapped within the earth. The kinetic energy $\Delta mc^2 \sim m_2c^2$ of this particle is then dissipated within the earth, and contributes to the earth's heat flow budget. The luminosity of the earth is approximately $44\mathrm{TW} \simeq 2.8 \times 10^{23}\mathrm{GeV\,s}^{-1}$, of which roughly half is accounted for by known mechanisms. So assuming a dark matter mass density near earth $\rho_{m;R_\oplus}$ with a velocity of $10^6\mathrm{cm\,s}^{-1}$, an earth geometric cross section of $4\pi(R_\oplus = 6.4 \times 10^8\mathrm{cm})^2$, and including [21] a solid angle acceptance factor of 1/2, we get the inequality

$$\frac{1}{2}\rho_{m;R_\oplus}c^2 10^6\mathrm{cm\,s}^{-1}4\pi(6\times 10^8\mathrm{cm})^2 \le \frac{1}{2}2.8\times 10^{23}\mathrm{GeV\,s}^{-1} \quad , \tag{35}$$

which gives the restrictive bound

$$\rho_{m;R_\oplus} \le 0.06(\mathrm{GeV}/c^2)\mathrm{cm}^{-3} \quad . \tag{36}$$

In the second case, the cross section for interaction of the secondary dark matter particle is very small, so that it escapes from the earth without interacting. In this case only the much smaller kinetic energy

$$\delta T_1 \sim m_1(\delta\vec{v}_1)^2/2 \tag{37}$$

of the recoiling nucleon is deposited in the earth. From (5), the ratio of this energy to Δmc^2 is of order

$$\frac{\delta T_1}{\Delta mc^2} \sim \frac{m_2}{2m_1} \quad , \tag{38}$$

which gives an effect that depends on the magnitude of m_2. For example, for m_2 of order 10 keV, (38) is of order 0.5×10^{-5}, and the bound of (36) altered now to

$$\rho_{m;R_\oplus} \le 10^4(\mathrm{GeV}/c^2)\mathrm{cm}^{-3} \quad . \tag{39}$$

[8] I wish to thank Susan Gardner and John Beacom for bringing this issue, and reference [21], to my attention.

Although the average effect of dark matter collisions is to alter the forward velocity of the flyby, there is also a random change in velocity that is averaged out in (7), which would show up as an increase in spacecraft temperature, as well as in possible localized structural disruption. Applying (37) in the flyby rest frame gives an estimate of the thermal energy gain by a nucleon per collision. Dividing this by the velocity gain by a nucleon per collision from (1)–(5), and multiplying by the total velocity change in the flyby (of order $\sim 10^{-6}|\vec{u}_1| \sim 1\,\mathrm{cm\,s^{-1}}$), gives the thermal energy gain by a nucleon in the course of the flyby,

$$\text{Temperature gain} \sim \frac{\delta T_1}{|\delta \vec{v}_1|} 10^{-6}|\vec{u}_1| \sim \frac{1}{2} m_1 |\delta \vec{v}_1| 10^{-6} |\vec{u}_1| \quad . \tag{40}$$

In the inelastic case, (5) gives $|\delta \vec{v}_1| \sim m_2 c/m_1$, so that (40) becomes

$$\text{Temperature gain} \sim \frac{1}{2} 10^{-6} m_2 |\vec{u}_1| c \sim 0.2^\circ\mathrm{K} \left(\frac{m_2 c^2}{\mathrm{MeV}} \right) \quad . \tag{41}$$

Similarly, in the elastic case, (4) gives $|\delta \vec{v}_1| \sim m_2 |\vec{u}_1 - \vec{u}_2|/m_1$, so that (40) gives

$$\text{Temperature gain} \sim \frac{1}{2} 10^{-5} m_2 |\vec{u}_1| |\vec{u}_1 - \vec{u}_2| \sim 10^{-5\circ}\mathrm{K} \left(\frac{m_2 c^2}{\mathrm{MeV}} \right) \quad . \tag{42}$$

These imply that the dark matter mass m_2 cannot be too large, or the temperature gain by the flyby would be noticeable; for example, from the inelastic case (41) we learn that the dark matter mass is constrained to be substantially less than a GeV. These results also suggest that sensitive calorimetry in high orbiting spacecraft, and perhaps even sensitive acoustic phonon detection, could be used to test the hypothesis that the flyby anomalies arise from earth-bound dark matter.

If the dark matter particles are too heavy, collisions with the spacecraft nucleons will cause recoils energetic enough to produce structural disruption. If we require that each individual nucleon recoil should not produce structural changes, then we get a condition of the form

$$\delta T_1 < E_{\mathrm{binding}} \quad , \tag{43}$$

with E_{binding} a characteristic atomic binding energy. In the inelastic case, where (38) with $\Delta m \sim m_2$ gives $\delta T_1 \sim m_2^2 c^2/m_1$, we then get the condition

$$m_2 c^2 < \left(m_1 c^2 E_{\mathrm{binding}} \right)^{1/2} \sim 100\mathrm{keV} \quad , \tag{44}$$

where for sake of illustration we have taken E_{binding} as 10 eV. Again, we see that dark matter particles, if responsible for the flyby anomalies, cannot be too massive.

In a steady-state situation, the dark matter particle capture at the earth's surface radius 6,400 km would have to be balanced by dark matter accumulation from solar system-bound dark matter, at or above the radius 70,000 km relevant for the flyby anomaly. Ignoring evaporation, which should be taken into account in a more careful estimate, and assuming similar dark matter velocities at radii 6,400 km and 70,000 km, this gives as the balance condition

$$\rho_{m;s.s.} f_{\rm e} 70^2 \sim \rho_{m;R_\oplus} 6.4^2 \quad , \tag{45}$$

which with $\rho_{m;s.s.} \leq 2 \times 10^2 ({\rm GeV}/c^2)\,{\rm cm}^{-3}$, and using $f_{\rm e} \leq 1$, gives the bound

$$\rho_{m;R_\oplus} \leq 2.4 \times 10^4 ({\rm GeV}/c^2)\,{\rm cm}^{-3} \quad . \tag{46}$$

This bound, from the steady state condition, is compatible with that of (39) obtained from the earth heat flow budget. We conclude that not only must the dark matter density be much smaller near the moon's orbit than at the radius relevant for the flyby anomaly, but it also must be substantially depleted near the earth's surface, to be consistent with estimates based on earth capture. Whether this depletion would extend to radii as large as the 30-40 thousand kilometer range relevant for high orbit satellites is not clear.

F. Summary

To summarize, we have made a preliminary survey of whether dark matter interactions can explain the flyby anomaly. Our estimates do not rule out this possibility (for example, we do not find a requirement that $f_{\rm e} >> 1$), but the constraints are severe. To explain the cases of negative drag flybys, exothermic inelastic scattering of dark matter on ordinary matter is required. The cases of positive drag require either elastic dark matter scattering, or an asymmetric dark matter velocity distribution in the inelastic case. In addition, the dark matter must be confined well within the moon's orbit and depleted near the earth's surface, a cascade accumulation mechanism is required to reach the needed dark matter density, the dark matter mass must be well below a GeV, the dark matter interaction cross section with nucleons must be relatively high (with an inelastic cross section lying between around $10^{-33}{\rm cm}^2$ and $10^{-27}{\rm cm}^2$), and dark matter must be non-self-annihilating. These constraints can be compatible, since for dark matter masses much below a GeV, there is little information on nucleon scattering cross sections,[9] Further detailed modelling will be needed to see whether the various constraints can be fulfilled.[10]

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[9] See figures 1.7-1.9 of [9], which summarize bounds from experiments searching for multi-GeV dark matter, and [4] for light dark matter fits to the DAMA/LIBRA signal. For dark matter self-interactions, an analysis [22] of the "bullet cluster" constrains the ratio of dark matter self-interaction cross section to mass to be $\sigma/m_2 < 0.7{\rm cm}^2{\rm g}^{-1}$, which corresponds to $\sigma < 10^{-29}{\rm cm}^2(m_2c^2/10{\rm keV})$.

[10] We also remark that for dark matter masses close to the electron mass, scattering from electrons can lead to much higher capture rates than scattering from nucleons with the same cross section, since the capture rate formula given

One could of course take the severity of the constraints as an indication that the flyby anomaly must be artifactual, and this may ultimately turn out to be the case. But if the anomaly is confirmed, and if the DAMA/LIBRA hints of light dark matter are also confirmed, then new physics[11] will be required, and the scenarios sketched here represent a possibility that merits further exploration.

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