Dark Matter in the Solar System

X. Xu* and E. R. Siegel†
Steward Observatory, University of Arizona, 933 N. Cherry Ave., Tucson, AZ 85721
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We determine the density and mass distribution of dark matter within our Solar System. We explore the three-body interactions between dark matter particles, the Sun, and the planets to compute the amount of dark matter gravitationally captured over the lifetime of the Solar System. We provide an analytical framework for performing these calculations and detail our numerical simulations accordingly. We find that the local density of dark matter is enhanced by between three and five orders of magnitude over the background halo density, dependent on the radial distance from the Sun. This has profound implications for terrestrial direct dark matter detection searches. We also discuss our results in the context of gravitational signatures, including existing constraints, and find that dark matter captured in this fashion is not responsible for the Pioneer anomaly. We conclude that dark matter appears to, overall, play a much more important role in our Solar System than previously thought.

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I. INTRODUCTION

Accounting for about 23% of the energy density of the Universe [1], dark matter is an integral part of our surroundings. It dominates the more familiar, baryonic components, which comprises only 4.4% of the Universe, on the largest scales. Achieving an understanding of this perplexing dark element is one of the most compelling unsolved problems in modern astrophysics.

The astrophysical evidence for the existence of dark matter is overwhelming, as observations of the cosmic microwave background [1], the power spectrum of the Universe [2], and colliding galaxy clusters [3] all point towards the same conclusion. With an understanding that a dark, pressureless, fluid-like component permeates the Universe, astrophysicists have successfully simulated the large-scale processes of structure formation in the context of a ΛCDM Universe [4]. More recently, some attention has been given to the dark matter substructure formed on subgalactic scales, down to scales of order 10^{-2} pc [5]. However, relatively little consideration has been given to the distribution of dark matter within our own Solar System.

Yet, dark matter may prove to be profoundly important in our Solar System for both its additional gravitational effects on planets and other orbiting bodies [6, 7, 8] as well as the motions of spacecraft [9, 10]. Furthermore, a knowledge of the density and velocity of dark matter particles is particularly important for terrestrial direct detection experiments [11].

In this paper, we model the Solar System and the dark matter that it encounters in order to quantify how much dark matter we expect the Solar System to have captured over its lifetime. Through favorable three-body gravitational interactions between a dark matter particle, the Sun, and any of the planets, a non-zero and possibly significant fraction of the dark matter passing through the Solar System will become gravitationally bound to it. The remainder of this paper is focused on solving this problem, and is laid out as follows: section 2 details the model chosen for the Solar System and galactic properties, and provides analytic details of our calculations. Section 3 sets forth the computations undertaken to successfully determine the probability of binding dark matter to the Solar System. Section 4 presents our results, including the density and mass distributions of dark matter with respect to distance from the Sun. We recommend that anyone not interested in the details of our calculations skip directly to section 4. Finally, section 5 concludes with a discussion of the conclusions reached from our analysis, detailing significant implications for dark matter detection and presenting a comparison of our results with the current experimental and observational constraints.

II. DARK MATTER CAPTURE

Our Sun (and hence our Solar System) is presently moving through the galaxy with well-known parameters [12] that have changed little, despite refinements in measurements [13], over many years. More recently, we have been able to determine that our Milky Way, like all comparable galaxies, is pervaded by a dark matter halo with a specific density profile [14]. N-body simulations also provide insights into modeling the Milky Way [15]. Based on the fact that our Sun is not an isolated body, but rather has eight planets as well as a number of other, less significant bodies gravitationally bound to it, a non-zero fraction of this dark matter will be captured by favorable three-body interactions between a dark matter particle, the Sun, and an orbiting body.

In order to compute the amount of dark matter cap-

*xxu@as.arizona.edu
†ethan@gravity.as.arizona.edu; http://startswithabang.com/

tured by the Solar System over the course of its 4.5 Gyr history, we assume the galactic parameters shown in Table I below. We then find that the Sun moves through

Parameter	Value
Oort's Constant A ($\mathrm{km\,s^{-1}\,kpc^{-1}}$)	14.4
Solar Radius $r_\odot$ (kpc)	8.0
Speed of Local Standard of Rest (LSR) $v_{\rm lsr}$	200
Sun's speed relative to LSR $v_{\odot,\rm lsr}$	20
Velocity relative to the Galactic Plane v_z	7
Solar Period relative to the Galactic Plane $t_{\rm osc}$ (Myr)	63
Local Dark Matter density $\rho_{\rm DM}(r_\odot)$ ($M_\odot\,\mathrm{pc}^{-3}$)	0.009
Mass within $r_\odot$ of Galactic center $M_{\rm enc}(r_\odot)$ ($10^{10}\,M_\odot$)	9
Local $rms-$velocity of Dark Matter $v_{\rm rms}(r_\odot)$	220

TABLE I: The local galactic and dark matter parameters used for the Milky Way and the present values of the Sun's distance and velocity components. Unless otherwise noted, all velocities are in units of $\mathrm{km\,s^{-1}}$.

the galaxy with a velocity ($v_\odot$) with components given in cylindrical coordinates:

$$\begin{aligned} v_{\odot,\rho} &= A r_\odot \cos{(2l)}, \\ v_{\odot,\phi} &= v_{\rm lsr} - v_{\odot,\rm lsr}\sin{(2l)}, \\ v_{\odot,z} &= v_z \sin\left(\frac{2\pi t}{t_{\rm osc}}\right), \end{aligned} \tag{1}$$

where l is the galactic longitude, t is a time coordinate, and the remaining parameters are defined in Table I. The individual dark matter particles are assumed to follow a Maxwell-Boltzmann distribution [16] with a probability distribution function $f(v)$ given by

$$f(v) = \sqrt{\frac{54}{\pi}}\frac{v^2}{v_{\rm rms}^3(r)} e^{-\frac{3}{2}\frac{v^2}{v_{\rm rms}^2(r)}}, \tag{2}$$

where the local rms-velocity $v_{\rm rms}(r_\odot)$ is given in Table I.

Therefore, the Sun sweeps out a predictable three-dimensional path over its 4.5 Gyr history. With the dark matter having a local density $\rho_{\rm DM}(r_\odot)$, a rms-velocity $v_{\rm rms}(r_\odot)$ and a velocity distribution as given above, the fraction of dark matter captured can be calculated in a straightforward fashion via modeling of the Solar System and the dark matter particles passing through it. However, the number of dark matter particles encountered is far too large and the rate of particle capture is far too small to effectively simulate using N-body methods. We are therefore forced to use analytic approximations to shape this problem into a tractable one.

We begin by considering an ensemble of dark matter particles at infinity each with a speed $|v_{\rm DM}|$ chosen from the Maxwell-Boltzmann distribution and a random orientation $\hat{v}_{\rm DM}$. We also consider the Sun, and a single planet with mass m_p, distance from the Sun r_p, and a circularized velocity around the Sun v_p. (This analysis will be repeated eight times, once for each of the planets.) We first choose a very large r_∞ to be the vector distance from the dark matter particle to the Sun, so that $|r_\infty| \gg |r_p|$, but small enough that when we compute the dark matter particle's angular momentum with respect to the Sun, we get a reasonable (i.e., non-infinite) value.

We then perform a coordinate transformation to shift into the Sun's rest frame, computing the velocity of the dark matter at infinity (v_∞) in that frame:

$$\begin{aligned} v_{\infty\parallel} &= \frac{(v_{\rm DM} - v_\odot)\cdot r_\infty}{|r_\infty|} \\ v_{\infty\perp} &= \frac{(v_{\rm DM} - v_\odot)\times r_\infty}{|r_\infty|}, \end{aligned} \tag{3}$$

where $v_{\infty\parallel}$ and $v_{\infty\perp}$ are the components of the dark matter particle's velocity parallel and perpendicular to the Sun's, respectively.

The dark matter particles we are interested in, as far as the possibility of gravitational capture goes, are those that pass within a distance r_p of the Sun. The dark matter particles, at infinity, will have an angular momentum L given by

$$L = m v_{\infty\perp} r_\infty, \tag{4}$$

where m is the mass of a dark matter particle. We note that this mass is completely unimportant in our analyses, as it does not enter into any of our equations; only the combination L/m appears. These particles are all in hyperbolic orbits around the Sun initially, with eccentricities ϵ given by

$$\epsilon = \sqrt{1 + \frac{v_\infty^2 (L/m)^2}{G^2 M_\odot^2}}. \tag{5}$$

We then find that any particle that meets the following condition will pass within a distance r_p of the Sun:

$$\epsilon \geq \left|\frac{(L/m)^2}{G M_\odot r_p} - 1\right|. \tag{6}$$

Upon encountering the planet, the particle may receive boosts (or anti-boosts) at two points during the interaction. The first occurs at entry into the sphere of radius r_p and the second occurs upon exit of the sphere. Taking both of these opportunities into account, we consider the approximation that the planet's position at any time is given by a random location on the sphere of radius r_p. We determine that of the particles that pass through the sphere of radius r_p, a fraction r_b^2/r_p^2 of those will gravitationally encounter the planet, where r_b equals

$$r_b \equiv 1.15 r_p \left(\frac{m_p}{M_\odot}\right)^{1/3}, \tag{7}$$

with r_b defined to be the radius of a "sphere of influence" of a smaller gravitational body relative to a larger one [17].

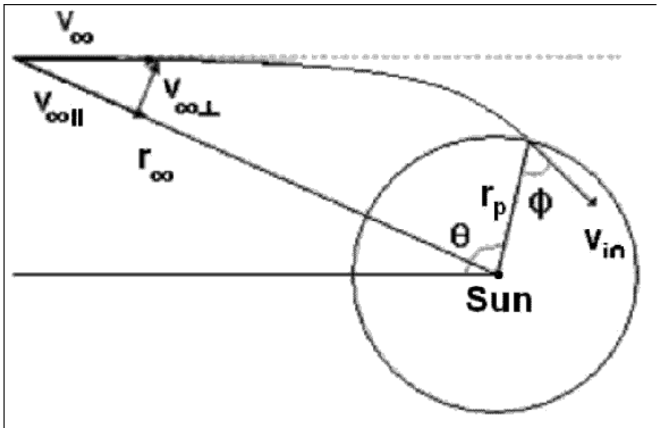

FIG. 1: Geometric setup for a dark matter particle approaching the Solar System from infinity in the Sun's rest frame. The semi-major axis of the planet in question's orbit is shown by r_p, with the velocity that the particle strikes the imaginary sphere of radius r_p given by $v_{\rm in}$. Through the conservation of angular momentum, the angle θ can be determined, as shown in equation (9), and the angle that the velocity $v_{\rm in}$ makes with r_p, defined as ϕ, is given by equation (10).

We then need to determine whether the gravitational encounter is favorable enough to transition the dark matter particle from an unbound, hyperbolic orbit around the Sun to a bound, elliptical one. We begin by determining the velocity of the incoming dark matter particle at a distance r_p from the Sun ($\equiv v_{\rm in}$). The geometry of an incoming dark matter particle as it enters the Solar System, possibly encountering a given planet's sphere of influence, is illustrated in Figure 1. This velocity will have a magnitude given by the conservation of energy,

$$|v_{\rm in}| = \sqrt{v_\infty^2 + \frac{2GM_\odot}{r_p}}, \tag{8}$$

and we are interested in determining the components parallel to the radial vector towards the Sun ($v_{\rm in\parallel}$) and perpendicular to it ($v_{\rm in\perp}$). We can determine these, in the Sun's rest frame, by first finding the angle of deflection (θ) caused by the gravitational force on the position of the dark matter particle's azimuthal angle from infinity to r_p,

$$\theta = -\cos^{-1}\left(\frac{\frac{(L/m)^2}{GM_\odot r_p} - 1}{\epsilon}\right) + \cos^{-1}\left(-\frac{1}{\epsilon}\right), \tag{9}$$

and then, through the conservation of angular momentum, the angle of deflection (ϕ) of the particle's velocity,

$$\phi = \sin^{-1}\left[\left|\frac{L/m}{v_{\rm in} r_p}\right|\left(\sin(\theta) \pm \cos(\theta)\sqrt{\frac{v_{\rm in}^2 r_p^2}{(L/m)^2} - 1}\right)\right], \tag{10}$$

where ϕ is chosen to be the smaller in magnitude of the two possible angles. From equations (8-10), we can then determine the components of $v_{\rm in}$ to be

$$v_{\rm in\parallel} = |v_{\rm in}\left[\sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi)\right]|, \tag{11}$$

$$v_{\rm in\perp} = \sqrt{v_{\rm in}^2 - v_{\rm in\parallel}^2}. \tag{12}$$

Assuming that the particle does get within a distance r_b of the planet (which, as stated above, it does with probability r_b^2/r_p^2), we then need to determine the gravitational effect of the planet on the dark matter particle's orbit with respect to the Sun. We assume that the planet is positioned randomly in space at a distance r_p from the Sun, and moves with a speed $|v_p|$ in a random direction perpendicular to $v_{\rm in\parallel}$.

We then switch to the planet's rest frame, and obtain, for the dark matter particle, a velocity ($v_{\rm dm}$) with components

$$v_{{\rm dm},x} = v_{\rm in\parallel}, \tag{13}$$

$$v_{{\rm dm},y} = v_{\rm in\perp} - v_p\sin(\alpha), \tag{14}$$

$$v_{{\rm dm},z} = -v_p\cos(\alpha), \tag{15}$$

where α is a random angle between 0 and 2π, and a position with respect to the planet ($r_{\rm dm}$) with components

$$r_{\rm dm} = \langle\sqrt{r_b^2 - r_{{\rm dm},y}^2 - r_{{\rm dm},z}^2}, r_{{\rm dm},y}, r_{{\rm dm},z}\rangle, \tag{16}$$

where $r_{{\rm dm},y}$ and $r_{{\rm dm},z}$ are randomly chosen to lie within a circle of radius r_b in the $yz-$plane. We also note that in this rest frame, the Sun appears to move with a velocity

$$v_\odot = \langle 0, -v_p\sin(\alpha), -v_p\cos(\alpha)\rangle. \tag{17}$$

We then compute the components of $v_{\rm dm}$ parallel and perpendicular to $r_{\rm dm}$,

$$v_{\rm dm\parallel} = \left|\frac{v_{\rm dm}\cdot r_{\rm dm}}{r_b}\right|, \tag{18}$$

$$v_{\rm dm\perp} = \left|\frac{v_{\rm dm}\times r_{\rm dm}}{r_b}\right|, \tag{19}$$

and the components of $v_\odot$ with respect to these new directions,

$$v_{\odot\parallel} = \frac{v_\odot\cdot r_{\rm dm}}{|r_b|}, \tag{20}$$

$$v_{\odot\perp} = v_\odot\cdot\left(\frac{v_{\rm dm}\times r_{\rm dm}}{|v_{\rm dm\perp}\, r_b|}\right), \tag{21}$$

$$v_{\odot 3{\rm d}} = \sqrt{v_\odot^2 - v_{\odot\parallel}^2 - v_{\odot\perp}^2}, \tag{22}$$

where $v_{\odot 3{\rm d}}$ is the component of $v_\odot$ orthogonal to both $\hat{v}_{\rm dm\parallel}$ and $\hat{v}_{\rm dm\perp}$. In this coordinate system, it is easy to compute the angle (β) that the planet causes the dark matter particle to deflect by,

$$\beta = 2\tan^{-1}\left|\frac{G\,m_p}{r_b\,v_{\rm dm\perp}\,v_{\rm dm}}\right|, \tag{23}$$

where the final outgoing velocity of the dark matter particle ($v_{\rm out}$) is then given by

$$\begin{aligned} v_{\rm out} &= \langle v_{\rm dm\parallel}\cos(\beta) + v_{\rm dm\perp}\sin(\beta), \\ &\quad -v_{\rm dm\parallel}\sin(\beta) + v_{\rm dm\perp}\cos(\beta), 0\rangle. \end{aligned} \tag{24}$$

At last, we can compute the total final speed of the dark matter particle ($|v_f|$), as it leaves the sphere of influence of the planet, in the rest frame of the Sun,

$$|v_f| = \sqrt{(v_{\rm out} - v_\odot)\cdot(v_{\rm out} - v_\odot)}. \tag{25}$$

The particle will be gravitationally captured if

$$v_f < v_{\rm escape}(r_p) \equiv \sqrt{\frac{2GM_\odot}{r_p}}, \tag{26}$$

and become bound in an elliptical orbit about the Sun with semi-major axis

$$a = \left(\frac{|v_f|^2}{GM_\odot} - \frac{2}{r_p}\right)^{-1}. \tag{27}$$

A point to note is that we do not consider any further three-body interactions between captured dark matter, the planets and the Sun. Once the dark matter is captured, we assume it remains captured without any further gravitational interactions of significance. However, such a back-reaction will exist, and could potentially decrease the amount of dark matter bound to the Solar System by a significant amount.

III. COMPUTATIONS

For each of the eight planets in our Solar System, we perform the calculations outlined in section II. The planets are assumed to have the parameters shown below in Table II. We begin by creating a cumulative distribution function for the Maxwell-Boltzmann distribution (equation (2)), obtaining cumulative probability $P(v)$ as a function of the dark matter's velocity,

Planet	Distance to Sun	Mass	Speed
Mercury	5.79×10^7 km	$1.68\times10^{-7}\,M_\odot$	$48\,{\rm km\,s^{-1}}$
Venus	1.08×10^8 km	$2.46\times10^{-6}\,M_\odot$	$35\,{\rm km\,s^{-1}}$
Earth	1.496×10^8 km	$2.99\times10^{-6}\,M_\odot$	$30\,{\rm km\,s^{-1}}$
Mars	2.28×10^8 km	$3.21\times10^{-7}\,M_\odot$	$24\,{\rm km\,s^{-1}}$
Jupiter	7.78×10^8 km	$9.50\times10^{-4}\,M_\odot$	$13\,{\rm km\,s^{-1}}$
Saturn	1.43×10^9 km	$2.86\times10^{-4}\,M_\odot$	$9.6\,{\rm km\,s^{-1}}$
Uranus	2.87×10^9 km	$4.40\times10^{-5}\,M_\odot$	$6.8\,{\rm km\,s^{-1}}$
Neptune	4.50×10^9 km	$5.11\times10^{-5}\,M_\odot$	$5.4\,{\rm km\,s^{-1}}$

TABLE II: Planet-Sun distances, planetary masses and speeds for the eight Solar System planets considered in this analysis. Distances are given in units of km, masses are given in units of solar masses, where $M_\odot = 1.9884\times10^{30}$ kg, and speeds in units of ${\rm km\,s^{-1}}$.

$$\begin{aligned} P(v) &= \int_0^v f(v')dv' \\ &= {\rm erf}\left(\sqrt{\frac{3}{2}}\frac{v}{v_{\rm rms}}\right) - \sqrt{\frac{6}{\pi}}\frac{v}{v_{\rm rms}}e^{-\frac{3}{2}\frac{v^2}{v_{\rm rms}^2(r)}}, \end{aligned} \tag{28}$$

where $P(v)$ is the probability of finding a dark matter particle with velocity less than or equal to v. From this cumulative distribution function, we determine the velocity with respect to the Sun and keep only those dark matter particles that meet the condition

$$|v_\infty| \le 2\left(v_p^2 + v_p\sqrt{\frac{2GM_\odot}{r_p}}\right)^{1/2}, \tag{29}$$

where v_p and r_p are the values for the appropriate planet as given in Table II, and v_∞ is the dark matter particle's speed with respect to the Sun's reference frame at infinity. We choose the condition in equation (29) because even the most favorable gravitational interaction with the planet can only decrease the speed of an incoming dark matter particle by $2v_p$. The condition in equation (29) is such that a particle reaching a distance r_p from the Sun will have a velocity no greater than $2v_p$. We generate at least one million unique particles that satisfy this condition for each planet.

Keeping track of the total number of particles simulated before generating the one million we seek, we compute the probability of particles satisfying the preceding speed constraint. We then generate random directions for a particle, with an initial position at r_∞, and a speed chosen randomly from the the set of one million particles. We simulate a total of 100 billion particles for each of the Jovian planets and from 300 to 600 billion particles for each of the inner, rocky planets, which require more due to their much smaller distances from the Sun. We adopt $r_\infty \equiv 2.67\times10^{15}$ m, placing it orders of magnitude beyond the orbit of Neptune, but still close enough so that a reasonable number of simulated particles will interact with each planet, based on the condition in equation (6). We then compute the location and velocity of all of the particles that do pass within a distance r_p of the Sun (correcting for the fact that the r_∞ chosen is not actually infinite), as well as the probability of randomly generated particles passing within r_p of the Sun.

We then reduce that probability further by a factor of r_b^2/r_p^2, as discussed in section II, as only that fraction of the dark matter particles passing within a distance r_p of the Sun will pass within a distance r_b of the planet in question.

We then generate, from the particles that have passed all the cuts up until now, a random set of positions for the planet within a three-dimensional distance r_b of the dark matter particle, according to equation (16), along with velocity directions for the planet in accordance with equations (13-15). From the particles that survive the earlier cuts, we choose enough random positions and velocity directions to generate one billion particles for this final step. By boosting to the planet's rest frame, calculating the change in direction due to the gravitational interaction between the dark matter particle and the planet, and then boosting back to the Sun's rest frame, we obtain the final speed of the particle. We tabulate the particles that become bound to the Sun as a result of this final interaction, and compute both the total probability of gravitational capture and the distribution of the semi-major axes of the captured particles.

IV. RESULTS

Given the r_∞ chosen in section III and the fact that the Solar System has had approximately 4.5 billion years (defined as the lifetime of the Solar System, $t_{\rm SS}$) to accrue dark matter via this mechanism, we determine that the total amount of dark matter encountered is given by

$$\begin{aligned} M_{\rm DM} &= \rho_{\rm DM}(r_\odot)\, V \\ &= \rho_{\rm DM}(r_\odot)\pi r_\infty^2 \bar{v}_\odot t_{\rm SS} \simeq 203\, M_\odot, \end{aligned} \qquad (30)$$

where $\bar{v}_\odot$ is the average velocity of the Sun with respect to the galaxy, determined to be $208\,{\rm km\,s^{-1}}$ using a time average of the data from equation (1) and Table I. Our

Planet	Fraction Captured	Mass Captured
Mercury	1.03×10^{-16}	$2.09 \times 10^{-14}\, M_\odot$
Venus	8.71×10^{-16}	$1.77 \times 10^{-13}\, M_\odot$
Earth	9.41×10^{-16}	$1.91 \times 10^{-13}\, M_\odot$
Mars	2.91×10^{-16}	$5.91 \times 10^{-14}\, M_\odot$
Jupiter	1.23×10^{-13}	$2.50 \times 10^{-11}\, M_\odot$
Saturn	7.06×10^{-14}	$1.43 \times 10^{-11}\, M_\odot$
Uranus	2.87×10^{-14}	$5.83 \times 10^{-12}\, M_\odot$
Neptune	3.98×10^{-14}	$8.08 \times 10^{-12}\, M_\odot$

TABLE III: Fraction and total mass of all dark matter particles captured due to gravitational interactions with each of the eight planets. Although the absolute numbers are much smaller for the inner, rocky planets compared to the Jovians, they are still significant for determining the densities of dark matter, as they dominate at radii smaller than half of Jupiter's semi-major axis.

calculations and computations then allow us to determine what fraction of this total mass winds up getting gravitationally captured via these three-body interactions. Our results are presented in Table III.

The two most important factors contributing to capture are the mass and the orbital semi-major axis of the planet. The more massive and the further away a planet is from the Sun, the more effective it will be at capturing dark matter, as the cross-section for favorable gravitational interactions is given by the radius of the sphere of influence (r_b) from equation (7).

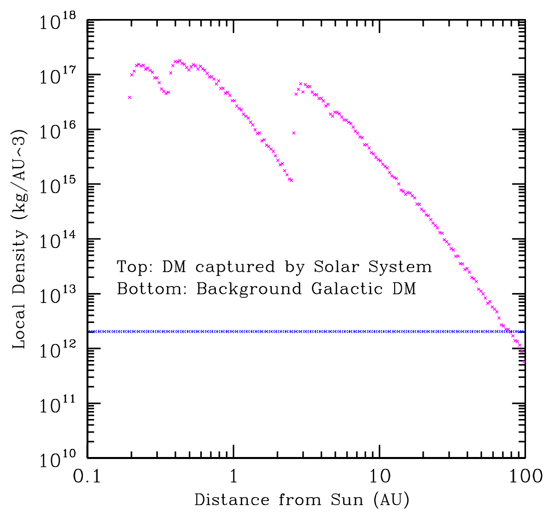

FIG. 2: Dark matter density vs. distance from the Sun in our Solar System at the present day. Density is presented in units of $\rm kg\,AU^{-3}$, with distances given in units of AU.

Equation (27) allows us to obtain a measure of the spatial distribution of the captured dark matter particles. In Figure 2, we illustrate the local density of dark matter, taking the semi-major axis of the captured dark matter particle as a proxy for its position, as a function of distance from the Sun. For comparison, the background galactic halo density is also shown. Of particular note is the dark matter density at 1 AU, which is greater than the background dark matter density (from the underlying galactic halo) by a factor of 1.63×10^4.

Figure 3 indicates the total mass of dark matter enclosed within a certain radius from the Sun due to both the captured dark matter and the underlying galactic halo. Between approximately 0.2 AU and 100 AU, the amount of dark matter bound to the Solar System is much more massive and dense than the background halo. Within the orbit of Mercury, Earth, Mars, and Neptune, we find that approximately 1.91×10^{16} kg, 3.23×10^{17} kg, 4.87×10^{17} kg, and 7.69×10^{19} kg of dark matter is enclosed, respectively. The total amount of matter dark bound to the Solar System as the result of gravitational capture is 1.07×10^{20} kg, or 1.78×10^{-5} Earth masses.

V. DISCUSSION

We predict the presence of a new component of dark matter within the Solar System due to gravitational capture. We find that, within the orbit of Neptune, 7.69×10^{19} kg of dark matter has become bound to our Solar System due to the capture mechanism over its life-

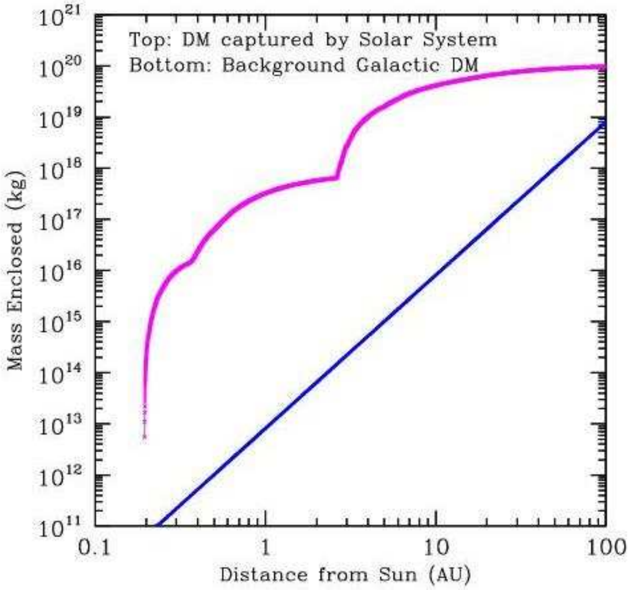

FIG. 3: Cumulatively enclosed mass as a function of distance from the Sun at present. This is representative of the additional mass gravitationally felt by an object in orbit around the Sun due to the presence of captured dark matter.

time. This is about a factor of 300 greater than the background mass from the galactic halo, as shown in Figure 3. The density of the captured dark matter is also significantly enhanced in comparison with the halo, as demonstrated in Figure 2. At the Earth's orbital radius, density is enhanced by more than four orders of magnitude over the local halo density, with a value of $3.3 \times 10^{16}\,\mathrm{kg\,AU^{-3}}$. Such elevated levels of dark matter have important implications for direct detection experiments and can be tested as a potential explanation for spacecraft anomalies.

Direct detection searches for dark matter rely predominantly on nuclear recoils [11], where the rate of interaction is dependent on the dark matter's density, velocity, and interaction cross-section (which itself may have a velocity dependence). Our determination of the local dark matter density and velocity distribution are of great importance to direct detection experiments. The most recent calculations that have been carried out assume that the properties of dark matter at the Sun's location are derived directly from the galactic halo [18]. By comparison, we find that terrestrial experiments should also consider a component of dark matter with a density 16,000 times greater than the background halo density, albeit with a v_{rms} on the order of the Earth's orbital speed ($30\,\mathrm{km\,s^{-1}}$), about a factor of seven smaller than the *rms*-velocity of the unbound halo particles. If this dark matter is not efficiently ejected by future interactions after the initial capture, the gravitationally bound component of dark matter may wind up dominating the signal expected from future direct detection experiments.

One method that has been used to constrain the amount of dark matter in the Solar System has been careful, long-term observations of planetary and satellite motions. Constraints have been obtained both from planetary orbital data [6, 8] and perihelion shift observations [7, 19]. The most stringent results for the dark matter density near Earth constrain ρ_{DM} to be less than $6.0 \times 10^{16}\,\mathrm{kg\,AU^{-3}}$ from orbital data [8]. For the densities near Mercury and Mars, perihelion precession provides the tightest constraints, yielding upper limits on ρ_{DM} of $8.7 \times 10^{18}\,\mathrm{kg\,AU^{-3}}$ near Mercury and $5.4 \times 10^{16}\,\mathrm{kg\,AU^{-3}}$ near Mars [19]. Our results satisfy these constraints, as we predict the densities near Mercury, Earth, and Mars to be $1.5 \times 10^{17}\,\mathrm{kg\,AU^{-3}}$, $3.3 \times 10^{16}\,\mathrm{kg\,AU^{-3}}$, and $8.5 \times 10^{15}\,\mathrm{kg\,AU^{-3}}$, respectively. Predictions about the effects of dark matter on planetary orbits are potentially observable, as the constraints on Earth and our predictions differ by less than a factor of two.

Another interesting issue to address is the Pioneer anomaly and the possibility that it has arisen due to the dark matter bound to our Solar System. Measurements show that Pioneer 10 and 11 have exhibited extra accelerations towards the Sun of $8.09 \pm 0.20 \times 10^{-8}\,\mathrm{cm\,s^{-2}}$ and $8.56 \pm 0.15 \times 10^{-8}\,\mathrm{cm\,s^{-2}}$, respectively [9]. In order for dark matter to have caused this, at least $3 \times 10^{-4} M_{\odot}$ of dark matter is required within 50 AU of the Sun [9]. The lower bound on the dark matter density capable of causing the anomalies is $6.0 \times 10^{18}\,\mathrm{kg\,AU^{-3}}$ for an inelastic scattering of dark matter particles [10]. Our results do not match either of these predictions, as we predict only $\sim 10^{20}\,\mathrm{kg}$ of dark matter enclosed within the entire Solar System and a dark matter density that never exceeds $2.0 \times 10^{17}\,\mathrm{kg\,AU^{-3}}$ anywhere. We conclude that the Pioneer anamoly cannot be caused by dark matter that has been captured by our Solar System.

Overall, we find that dark matter in our Solar System is far more important than previously thought. Due to gravitational three-body interactions between dark matter particles, the Sun, and the planets, a significant amount of dark matter winds up gravitationally bound to our Solar System, resulting in density enhancements between two and five orders of magnitude, depending on the distance from the Sun. A future direction for this work is to include back-reaction effects, such as subsequent gravitational scatterings of the captured particles. These may prove to be important in ejecting a portion of the captured dark matter particles, and in reducing the total amount of dark matter that remains bound to our Solar System. More accurate modeling of our galaxy's dark matter halo, such as the possible inclusion of a dark matter disk in the plane of our galaxy [20], will also alter the net amount of dark matter captured, and is worth further study. Our results may lead to exciting new directions in direct detection experiments and our understanding of dark matter on the smallest scales.

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